Examples of 2D geometric figures in SMath Studio

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This worksheet illustrates the use of SMath Studio 2D graphs to produce selected regular and irregular geometric figures.

Straight-line segment:

Given the two points, A and B, representing the extremes of the straight-line segment, plot the segment.

* Extreme points: $A := \begin{pmatrix} -5 \\ -2 \end{pmatrix}$ $B := \begin{pmatrix} 3 \\ 10 \end{pmatrix}$

* Generate matrix of points:

  * Slope: $m := \frac{B_2 - A_2}{B_1 - A_1}$ $m = 1.5$

  * Intercept: $b := A_2 - m \cdot A_1$ $b = 5.5$

  * Number of points: $n := 5$

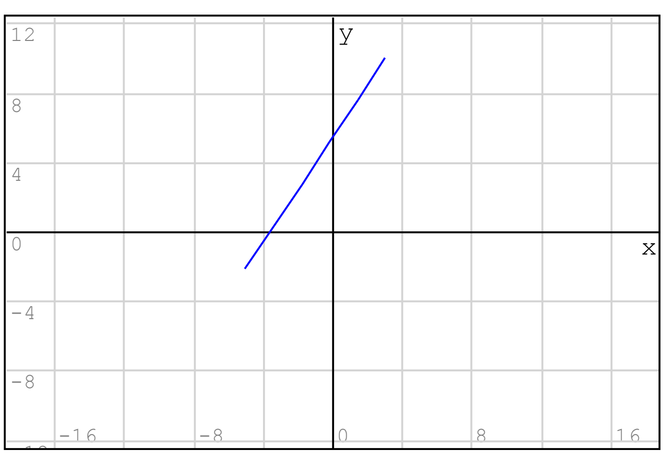
  * Increment: $\Delta x := \frac{B_1 - A_1}{n}$ $\Delta x = 1.6$

  * x-series: $xS := A_1 , A_1 + \Delta x .. B_1$

  * y-series:

    for $k \in 1 .. n + 1$
    $yS_k := m \cdot xS_k + b$

  * Matrix of points: $MS := \text{augment}(xS , yS)$



MS

* Mid-point: $M := \frac{1}{2} \cdot (A + B)$ $\qquad M = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

* Normal slope: $mP := -\frac{1}{m}$ $\qquad mP = -0.6667$

* Normal intercept: $bP := M_2 - mP \cdot M_1$ $\qquad bP = 3.3333$

* Normal increment: $\Delta xP := \frac{B_2 - A_2}{n}$ $\qquad \Delta xP = 2.4$

* Normal x series: $xP := A_2, A_2 + \Delta x .. B_2$

* Normal y-series:

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for k ∈ 1 .. n+1
   yP_k := mP· xS_k + bP
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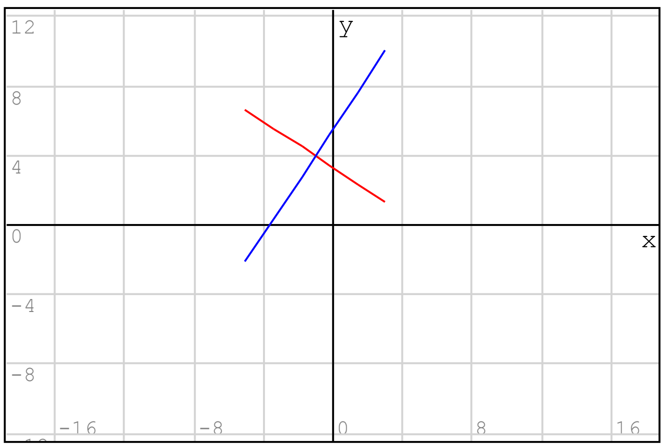
* Matrix of points (normal): $MP := \mathrm{augment}(xS, yP)$



MS
MP

Circle:

Given the center, C, and radius, r, of a circle, plot the circle.

* Center: $C := \begin{pmatrix} -3 \\ 5 \end{pmatrix}$ $\qquad$ * Radius: $r := 12.5$

* Generate matrix of data: $n := 150$

$\Delta\theta := \frac{2 \cdot \pi}{n}$ $\qquad \Delta\theta = 0.0419$

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for k ∈ 0 .. n
   | xC_(k+1) := C_1 + r· cos(k· Δθ)
   | yC_(k+1) := C_2 + r· sin(k· Δθ)
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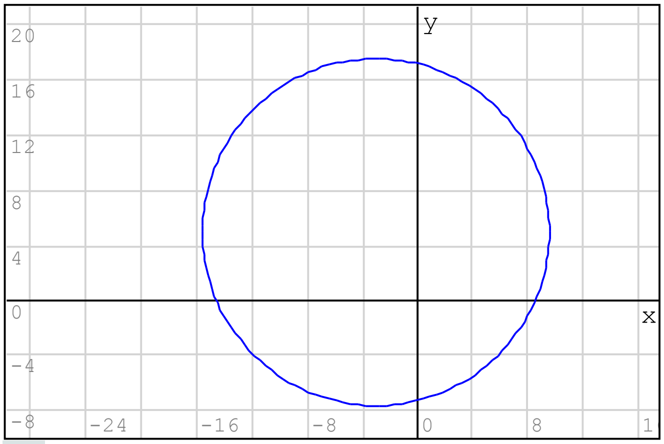
$MC := \mathrm{augment}(xC, yC)$



MC

Regular polygon of n sides:

Given the number of sides, n, the center of the circums-
cribed circle, C, and its radius, plot the regular polygon
of n sides. This example shows a regular pentagon.

* Number of sides: $n := 5$

* Center of polygon: $C := \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

* Radius of circums- cribed circle: $r := 12$

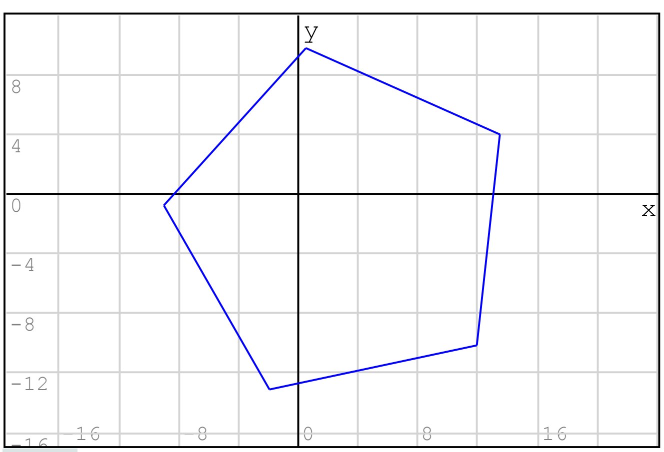
* Initial angle: $\theta 0 := \frac{\pi}{6}$

* Incremental angle: $\Delta\theta := \frac{2 \cdot \pi}{n}$ $\Delta\theta = 1.2566$

* Generate matrix of points:

$$\text{for } k \in 0..n \quad \begin{aligned} xPol_{k+1} &:= C_1 + r \cdot \cos(\theta 0 + k \cdot \Delta\theta) \\ yPol_{k+1} &:= C_2 + r \cdot \sin(\theta 0 + k \cdot \Delta\theta) \end{aligned}$$

$MPol := \text{augment}(xPol, yPol)$



MPol

Calculating the area and perimeter of the regular polygon:

The matrices representing the vertices of a polygon, whether regular or not, include an extra repetition of the first vertex:

$$MPol = \begin{pmatrix} 13.3923 & 4 \\ 0.5051 & 9.7378 \\ -8.9343 & -0.7457 \\ -1.8808 & -12.9625 \\ 11.9177 & -10.0296 \\ 13.3923 & 4 \end{pmatrix}$$

With such a matrix available, the area and perimeter of the polygon are calculated as follows.

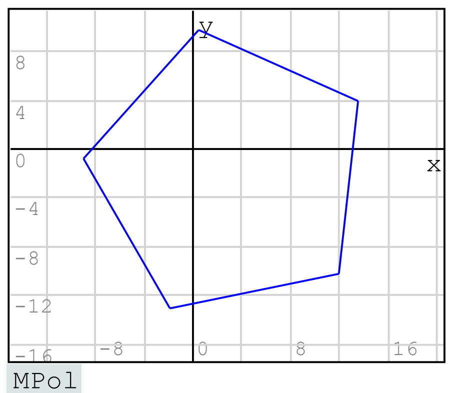
$M := MPol$

$n := rows(M) - 1$ $\quad\quad$ $n = 5$

$Area := 0$

$Perim := 0$

$$\begin{aligned}
&\text{for } k \in 1..n \\
&\quad Area := eval\left(Area + \frac{1}{2}\cdot\left(M_{k\,1}\cdot M_{k+1\,2}\right)\right) \\
&\quad Area := eval\left(Area - \frac{1}{2}\cdot\left(M_{k+1\,1}\cdot M_{k\,2}\right)\right) \\
&\quad d2 := eval\left(\left(M_{k+1\,1} - M_{k\,1}\right)^2\right) \\
&\quad d2 := eval\left(d2 + \left(M_{k+1\,2} - M_{k\,2}\right)^2\right) \\
&\quad Perim := eval\left(Perim + \sqrt{d2}\right) \\
&\quad Area := |Area|
\end{aligned}$$



$Area = 342.3803$ $\quad\quad$ $Perim = 70.5342$

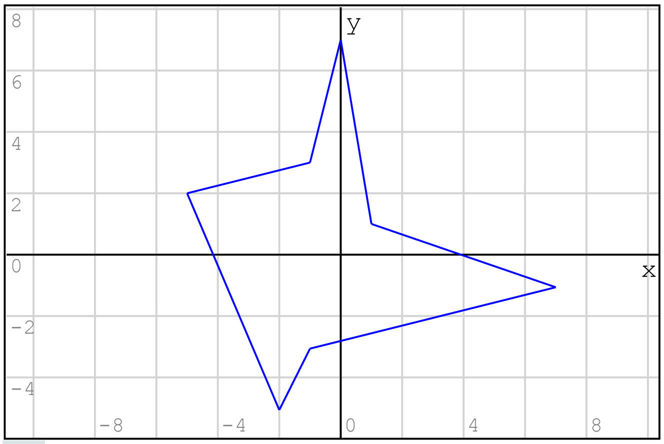
Irregular polygon:

Enter the (x,y) coordinates of the n polygon vertices in a nx2 matrix, e.g.,

$$MI := \begin{pmatrix} -5 & 2 \\ -1 & 3 \\ 0 & 7 \\ 1 & 1 \\ 7 & -1 \\ -1 & -3 \\ -2 & -5 \end{pmatrix}$$

Typically, the initial point needs to be repeated for the polygon to be completed. This is accomplished by using:

$$MN := augment\left(MI^T, row(MI, 1)^T\right)^T$$



MN

$$MN = \begin{pmatrix} -5 & 2 \\ -1 & 3 \\ 0 & 7 \\ 1 & 1 \\ 7 & -1 \\ -1 & -3 \\ -2 & -5 \\ -5 & 2 \end{pmatrix}$$

Calculating the area and perimeter of the irregular polygon:

As with the case of the regular polygon, the matrix representing the vertices of the irregular polygon include an extra repetition of the first vertex:

$$MN = \begin{pmatrix} -5 & 2 \\ -1 & 3 \\ 0 & 7 \\ 1 & 1 \\ 7 & -1 \\ -1 & -3 \\ -2 & -5 \\ -5 & 2 \end{pmatrix}$$

With such a matrix available, the area and perimeter of the polygon is calculated as follows.

Calculating area and perimeter for MN:

$M := MN$

$n := rows(M) - 1$ $\qquad$ $n = 7$

$Area := 0$

$Perim := 0$

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for k ∈ 1 .. n
  | Area := eval(Area + 1/2·(M[k,1]·M[k+1,2]))
  | Area := eval(Area − 1/2·(M[k+1,1]·M[k,2]))
  | d2 := eval((M[k+1,1] − M[k,1])^2)
  | d2 := eval(d2 + (M[k+1,2] − M[k,2])^2)
  | Perim := eval(Perim + √d2)
  | Area := |Area|
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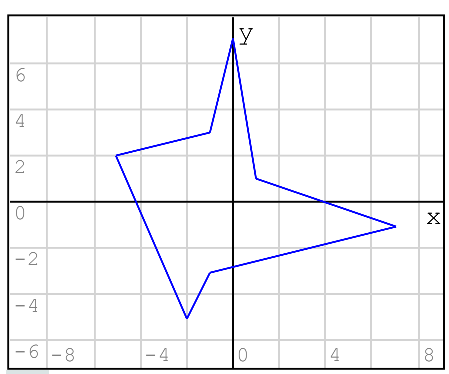

MN

$Area = 7.5$ $\qquad$ $Perim = 38.7516$
