//Density function - Normal distribution

$\mu := 1 \qquad \sigma := 0.4$

$$f(x,\mu,\sigma) := \frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\cdot e^{-\frac{1}{2}\cdot\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$df(x,\mu,\sigma) := \frac{d}{dx}f(x,\mu,\sigma)$$ //first derivative

$$df2(x,\mu,\sigma) := \frac{d^2}{dx^2}f(x,\mu,\sigma)$$ //second derivative

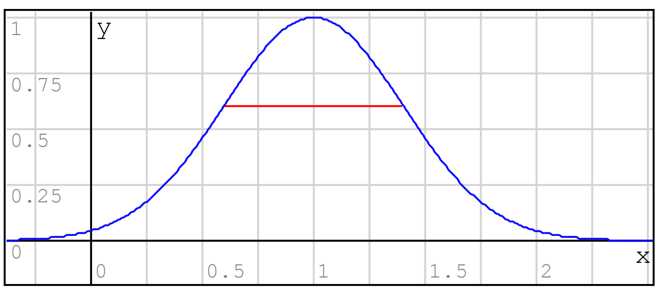
$modal := solve(df(x,\mu,\sigma) = 0, x, -2, 2)$

$modal = 1 \qquad df2(modal,\mu,\sigma) = -6.2335$ //Modal - maximal value

$infl := solve(df2(x,\mu,\sigma) = 0, x, -2, 2)$ //Inflection points

$infl = \begin{pmatrix} 0.6 \\ 1.4 \end{pmatrix} \qquad fi := \begin{pmatrix} f(infl_1,\mu,\sigma) \\ f(infl_2,\mu,\sigma) \end{pmatrix} \qquad ff := augment(infl, fi)$

$f1(x) := f(x,\mu,\sigma)$



$\begin{cases} f1(x) \\ ff \end{cases}$

$infl = \begin{pmatrix} 0.6 \\ 1.4 \end{pmatrix}$

$\mu \pm \sigma = \begin{cases} 1.4 \\ 0.6 \end{cases}$

//Calculating probabilities for μ ± zσ, where z=1,2,3

$$\int_{\mu-\sigma}^{\mu+\sigma} f1(x)\,dx = 0.6827$$

$$\int_{\mu-2\cdot\sigma}^{\mu+2\cdot\sigma} f1(x)\,dx = 0.9545$$

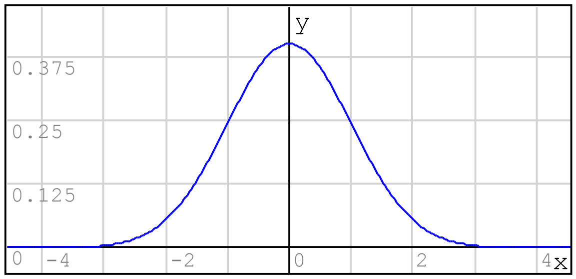
$$\int_{\mu-3\cdot\sigma}^{\mu+3\cdot\sigma} f1(x)\,dx = 0.9973$$

//Standardized Normal distribution

$\mu := 0 \qquad \sigma := 1$

//density function

$f1(x) := f(x,\mu,\sigma)$



f1(x)

//How to find the inverse problem. Find the z-value
//for the given probability

//Here are standard values of probabilities used in statistics

$p90 := 0.9 \quad p95 := 0.95 \quad p99 := 0.99$

//The density function is simetrical and we are
//finding only the positive value

$f(z) := \frac{p90}{2} - \int_0^z f1(t)\,dt$ //put p95, p99 instead of p90

$maxiter := 20 \quad \delta := 0.5 \cdot 10^{-6} \quad konv := maxiter \cdot \delta$

$x1 := 2 \quad x2 := 1.01 \cdot x1$

$i := 1$

while $(|konv| > \delta) \wedge (i \le maxiter)$

$$x := \frac{f(x2) \cdot x1 - f(x1) \cdot x2}{f(x2) - f(x1)}$$

$$konv := \frac{x - x2}{x + \delta}$$

$x1 := x2$

$x2 := x$

$i := i + 1$

$i = 8 \quad konv = -2.486 \cdot 10^{-10}$ $x = 1.6449$ //Well known z-value for
//P(-z<Z<z)=0.90

//for p95 the result will be x=1.96, z value for P(-z<Z<z)=0.95

//for p99 the result will be x=2.5758, z value for P(-z<Z<z)=0.99

//END

Or, you have to copy two more times

$f(z) := \frac{p95}{2} - \int_0^z f1(t)\,dt$ //put p95, p99 instead of p90

$maxiter := 20 \quad \delta := 0.5 \cdot 10^{-6} \quad konv := maxiter \cdot \delta$

$x1 := 2 \quad x2 := 1.01 \cdot x1$

$i := 1$

while $(|konv| > \delta) \wedge (i \le maxiter)$

$$x := \frac{f(x2) \cdot x1 - f(x1) \cdot x2}{f(x2) - f(x1)}$$

$$konv := \frac{x - x2}{x + \delta}$$

$x1 := x2$

$x2 := x$

$i := i + 1$

$i = 5$ $\quad konv = -1.8549 \cdot 10^{-7}$ $\quad x = 1.96$

$$f(z) := \frac{p99}{2} - \int_0^z f1(t)\,dt$$

//put p95, p99 instead of p90

$maxiter := 20$ $\quad \delta := 0.5 \cdot 10^{-6}$ $\quad konv := maxiter \cdot \delta$

$x1 := 2$ $\quad x2 := 1.01 \cdot x1$

$i := 1$

while $(|konv| > \delta) \wedge (i \le maxiter)$

$$x := \frac{f(x2) \cdot x1 - f(x1) \cdot x2}{f(x2) - f(x1)}$$

$$konv := \frac{x - x2}{x + \delta}$$

$x1 := x2$

$x2 := x$

$i := i + 1$

$i = 8$ $\quad konv = 2.9524 \cdot 10^{-7}$ $\quad x = 2.5758$
